Unequal Equals

5.1

Solving Polynomial Inequalities

LEARNING GOALS

In this lesson, you will:

- Determine all roots of polynomial equations.
- Determine solutions to polynomial inequalities algebraically and graphically.

Income Inequality is a term used to describe the gap or difference between the amount of money that wealthy people possess as compared to the amount of people without wealth. From the 1950s through the 1970s, the trend in the United States was toward *more income* equality. In other words, non-wealthy people earned money at a faster rate than the wealthiest segment of the population, creating a smaller gap between these two social classes. Many economists attribute this trend towards equality to industrial boom leading up to and following World War II. Millions of soldiers returning from active war duty after World War II received low interest loans for housing, and money for college and career-training. This helped non-wealthy people earn a greater share of the country's wealth. In the 1970s the wealthiest 1% of the population owned approximately 9% of America's total wealth.

Since the 1970s, the United States has become a nation with much more income inequality. Wages in the middle and lower classes have remained fairly stagnant while the wealth of the top 1% has increased from 9% in the 1970s to nearly 25% today.

Why do you think the income inequality changed after the 1970s? Do you think this trend will continue for the foreseeable future? What factors play a part in determining wealthy and non-wealthy classes?

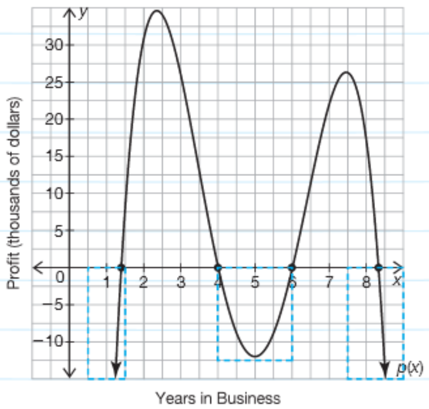
PROBLEM 1 Analyzing Profits

Lawn Enforcement is a small landscaping company. It has a profit model that can be represented by the function,

$$p(x) = -x^4 + 19.75x^3 - 133.25x^2 + 351.25x - 280.75$$

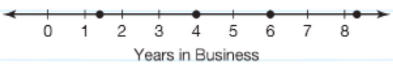
where profit, in thousands of dollars, is a function of time, in years, the company has been in business. Let's analyze $p(x)$ represented on a graph.

The graph shown represents the change in profit as a function of the number of years that Lawn Enforcement has been in business.



The points identified on the graph represent the zeros of the function where Lawn Enforcement's profit was 0.

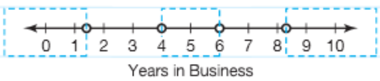
Each point on the number line represents the years in business when Lawn Enforcement's profit was 0.



The function $p(x) = 0$ when $x = 1.4, 4, 6, 8.3$.

The regions enclosed in dashed boxes on the coordinate plane represent Lawn Enforcement's profit less than 0.

The regions on the number line enclosed in dashed boxes represent the years in business when Lawn Enforcement's profit was less than 0.



The function $p(x) < 0$ when $\left\{\begin{array}{c} x < 1.4 \\ 4 < x < 6 \\ x > 8.3 \end{array}\right\}$.

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1. Analyze the worked example.

   a. Why were the points changed to open circles on the number line to represent the years in business when $p(x) < 0$.

   b. Circle the parts of the graph on the coordinate plane that represent where $p(x) > 0$. Then circle the intervals on the number line that represent the years in business where $p(x) > 0$. Finally identify the set of x-values to complete the sentence and explain your answer in terms of this problem situation.

   The function $p(x) > 0$ when ____________________.

   c. Draw a solid box around the segment(s) where $p(x) > 35{,}000$. Then identify the set of x-values to complete the sentence. Finally, explain your answer in terms of this problem situation.

   The function $p(x) > 35{,}000$ when ______________________.

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PROBLEM 2 Analyzing Methods for Solving Polynomial Inequalities

In this lesson, you will solve polynomial inequalities, which are very similar to solving linear inequalities. Recall from your experience of solving linear inequalities graphically, that < or > is represented with a dotted line, and ≤ or ≥ is represented with a solid line. Also remember that when you are determining which region(s) to shade, look at *y*-values above or below the boundary line depending on the inequality sign. It is always a good idea to check your work by selecting test points as well.

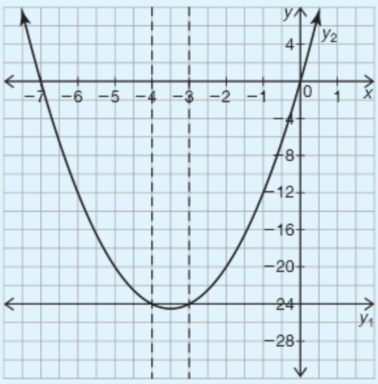
1. Samson, Kaley, Paco, and Sal each solved the quadratic inequality $-24 > 2x^2 + 14x$.

Samson

I graphed both sides of the inequality.

$y_1 = -24$

$y_2 = 2x^2 + 14x$



I drew vertical dashed lines at the two points where the graphs intersect.

I can then determine from the graph that the *x*-values of $2x^2 + 14x$ that generate values less than -24 are between -4 and -3.

Therefore the solution to the inequality is $-4 < x < -3$.

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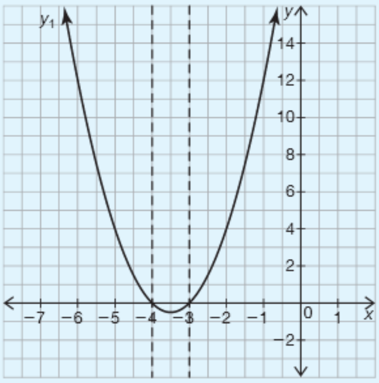
Paco

I added 24 to both sides of the inequality because I wanted one side to be equal to 0. Then, I graphed that inequality.

$y_1 = 2x^2 + 14x + 24$



I drew vertical dashed lines where the graph crosses the x-axis.

I can then determine from the graph that the x-values of $2x^2 + 14x$ that generate values less than 0 are between −4 and −3.

Therefore the solution to the inequality is $-4 < x < -3$.

a. Explain why the graphs of Samson and Paco are different, yet generate the same answers.

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b. Explain the error in Sal's work.

Kaley

I remember from solving linear inequalities that I can first treat the inequality as an equation and solve:

$0 = 2x^2 + 14x + 24$

$0 = 2(x^2 + 7x + 12)$

$0 = 2(x + 3)(x + 4)$

$x = -4, -3$

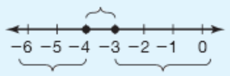
This means that the x-intercepts are -4 and -3, breaking up the number line into 3 parts and testing each section in the original inequality $-24 > 2x^2 + 14x$, I can determine the solution:

Test $x = -3.5$

$-24 > 2(-3.5)^2 + 14(-3.5)$

$-24 > -24.5$

✓



Test $x = -5$

$-24 > 2(-5)^2 + 14(-5)$

$-24 > -20$

✗

Test $x = 0$

$-24 > 2(0)^2 + 14(0)$

$-24 > 0$

✗

The only section that satisfies the original inequality is when x is between -4 and -3 so the solution to the inequality is $-4 < x < -3$.

Sal

I remember from solving linear inequalities that I can treat the inequality as an equation and solve:

$2x^2 + 14x = -24$

$2x(x + 7) = -24$

$2x = -24 \quad (x + 7) = -24$

$x = -12 \quad x = -31$

This means that the x-intercepts are -12 and -31, so the solution to the inequality is $-31 < x < -12$.

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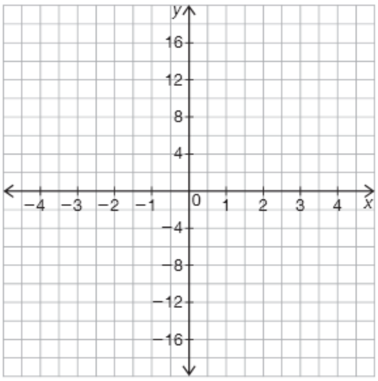
c. Compare Samson's method to Kaley's method. List advantages and disadvantages of each method.

2. Solve $18 \leq 3x^2 + x$ using any method. Explain why you chose the method.



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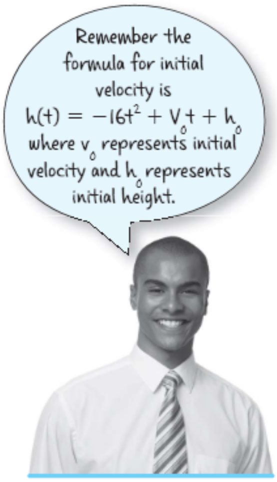
PROBLEM 3 Real Life Kids

Polynomial inequalities can be used to represent everyday situations. Write and solve each real-world inequality.

1. Get Your Kicks is an indoor soccer complex. The roof's height at the facility is 80 feet. If a soccer ball is kicked and touches the ceiling during a game, the team that kicked the ball must have a player sit out for two minutes. Michael kicks a ball straight up in the air with an initial velocity of 73 feet per second.

   a. Write an inequality to represent this problem situation.

   b. Use your inequality to determine whether Michael's team will be penalized for hitting the ceiling. Explain your reasoning.



2. Glen High School's student council is hosting a dance to raise money for panda bears. The dance will cost $2250. At the current ticket price of $10, the council knows that they will have 185 people attend the dance. This is not enough people to cover the cost of the dance, so they estimate that for every $0.25 decrease in ticket price, 15 more people will attend the dance.

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   a. Write an equation that will represent the profit that the dance will make.

   b. Write an inequality to represent the dance making a profit.

c. Determine the maximum price the council can charge for tickets and still make a profit.

d. Determine the price of the ticket that will maximize profit. What is the maximum profit?

3. Use a graphing calculator to solve each inequality.

a. $-5 \geq x^3 - 9x$

b. $0 < 2x^3 - 3x^2 - 3x + 2$

4. The average blood sugar (also known as glucose) levels in a person's blood should be between 70 and 100 mg/dL (milligrams per deciliter) one hour after eating. A person with Type 2 diabetes strives to keep glucose levels under 120 mg/dL with diet and exercise in order to avoid insulin injections. Glucose levels of one individual over the span of 72 hours can be represented with the polynomial function,

$$b(t) = 0.000139x^4 - 0.0188x^3 + 0.8379x^2 - 13.55x + 176.51$$

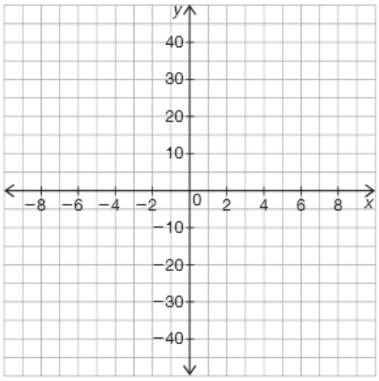
where glucose levels is a function of the number of hours.

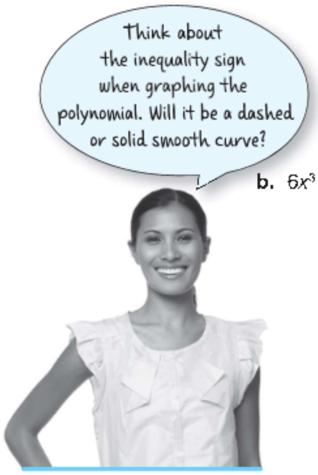
a. For what hours were the glucose levels greater than 120 mg/dL?

b. For what hours were the glucose levels less than 120 mg/dL?

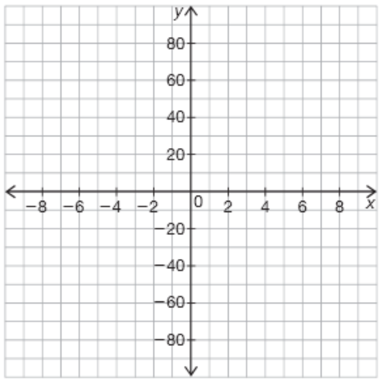
5. Solve each inequality by factoring and sketching. Use the coordinate plane to sketch the general graph of the polynomial in order to determine which values satisfy the inequality.

a. $2x^3 - 8x^2 - 8x + 32 > 0$





b. $6x^3 - 21x^2 - 12x > 0$



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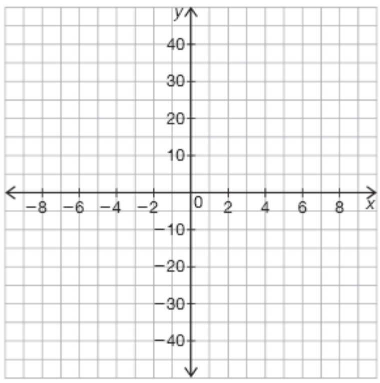
c. $x^4 - 13x^2 + 36 \leq 0$



Be prepared to share your solutions and methods.